

Lecture 8: fibered categories 3

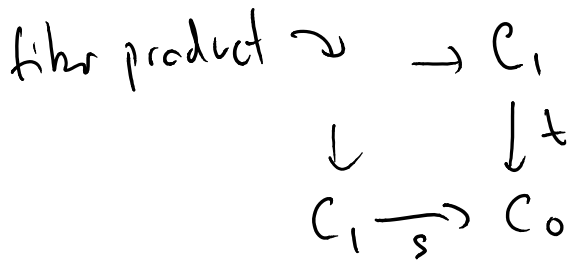
Thursday, September 11, 2014 11:02 AM

Category: Sets $\mathcal{C} = (\mathcal{C}_0, \mathcal{C}_1)$
 objects \uparrow \downarrow arrows

$$s, t: \mathcal{C}_1 \rightarrow \mathcal{C}_0 \quad (\text{source : target})$$

$$\varepsilon: \mathcal{C}_0 \rightarrow \mathcal{C}_1 \quad (\text{identity})$$

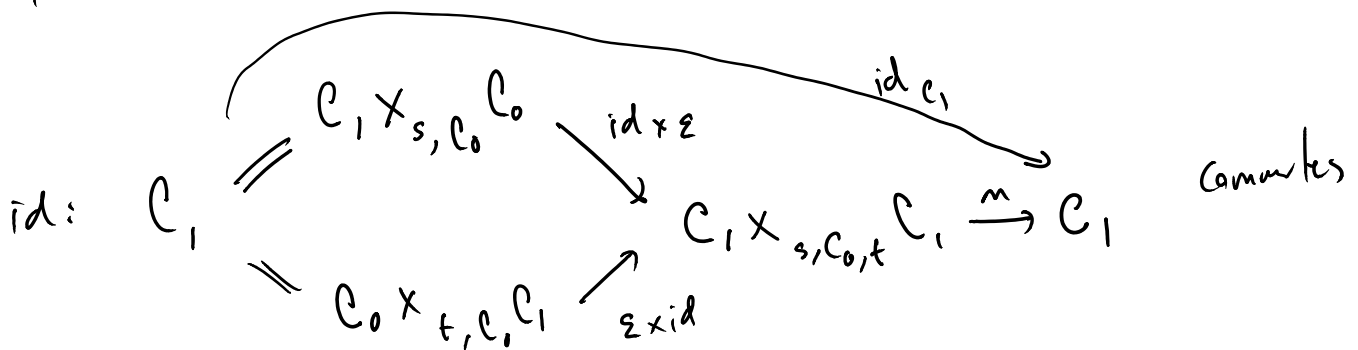
$$m: \underbrace{\mathcal{C}_1 \times_{s, \mathcal{C}_0, t} \mathcal{C}_1}_{\text{fiber product}} \rightarrow \mathcal{C}_1$$



s.l.

ε is id

start & end: $s \circ \varepsilon = t \circ \varepsilon = \text{id}_{\mathcal{C}_0}$, $s \circ m = s \circ \text{pr}_2$, $t \circ m = t \circ \text{pr}_1$ maps from fiber prod



assoc:

$$C_1 \times_{s, C_0, t} C_1 \times_{s, C_0, t} C_1 \xrightarrow{\text{id} \times m} C_1 \times_{s, C_0, t} C_1 \xrightarrow{m} C_1$$

commutes.

Given a cat \mathcal{C} , a cat. object in \mathcal{C} is a pair of objects C_0, C_1 w/ morphisms s, t, ε, m as above.

Similarly, groupoid objects in a category \mathcal{C} as above, but w/ extra morphism $i: C_1 \rightarrow C_1$ "inverse"

Groupoid = groupoid object in Set

eq. rel: $R \overset{i}{\subset} S \times S$ $(s, t) \in R \implies s \sim t$ (well def mult)

$$C_1 \overset{\text{pr}_{0i}}{\underset{\text{pr}_{2i}}{\rightrightarrows}} S \overset{i}{\subset} C_0$$

$s \sim t \wedge u \implies s \sim u$
 $s \sim s \implies s \sim t \implies t \sim s$
 $\uparrow \qquad \qquad \uparrow$
 id inverses

note: if \exists arrow $s \rightarrow t$ then it is unique.
 \implies cat \rightsquigarrow groupoid.

alternately eq. rel = gpoid w/ all "arrows unique"

$$\forall a \in C_0, \text{Aut}_C(a) = \text{id}_a$$

alternately: Cat is an eq. rel if it is eq. to a rel.

exercise \rightarrow

Given a category C & a groupoid object $X = (X_0, X_1, \dots)$ in C , we get a cat fibered in groupoids (C) !

(Recall if $Y \in C$, (C/Y) fibred cat)

$$\begin{array}{ccc} \{X_0/X_1\} & \text{for } u \in C, & \{X_0/X_1\}(u) = (X_0(u), X_1(u), \dots) \\ \downarrow & & \uparrow \text{groupoid} \\ C & & X_0(u) = \text{Hom}(u, X_0) \dots \end{array}$$

if given $V \xrightarrow{f} u$

get $\text{Hom}(u, X_0) \xrightarrow{f^*} \text{Hom}(V, X_0)$

$$\{X_0/X_1\}$$

objects are $(u, u) \quad u \in C, u \in X_0(u)$

morphisms

$$\text{Hom}((u, u), (V, v)) =$$

map $f: U \rightarrow V$; an isom
 $(X, (u))$
 $u \rightarrow f^*v$

F
 $\downarrow P$
 \mathcal{C}

shared cat, $x, x' \in F(X)$

$$\underline{\text{Isam}}(x, x') : (\mathcal{C}/X)^{\text{op}} \rightarrow \text{"Set"}$$

$$\underline{\text{Isam}}(x, x')(X) = \text{Hom}_{F(X)}(x, x')$$

$$\underline{\text{Isam}}(x, x') \ni (h, u), f: y \xrightarrow{\sim} y' \in F(u)$$

$u \rightarrow x$
 y

where y is a pullback of x
 over u

$$\downarrow$$

$$(\mathcal{C}/X)$$

$$\begin{array}{ccc} y & \longrightarrow & x \\ \downarrow & & \downarrow \\ u & \xrightarrow{f} & x \end{array}$$

y' pullback of x'

morphism

$$\begin{array}{ccc} y & \xrightarrow{f} & y' \\ \downarrow F(u) & & \downarrow F(u) \\ z & \xrightarrow{g} & z' \end{array}$$

f, u

\downarrow

g, u

but Isam is a functor!